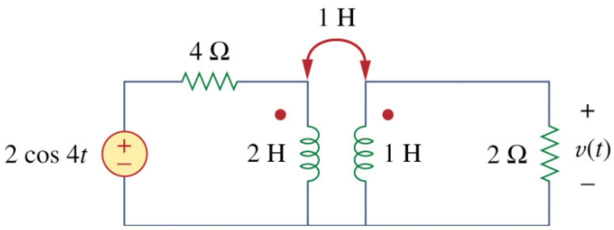
This is the last homework assignment. Some of the content will require the lectures of December 4.

1:

Find $v(t)$ for the circuit below.



Let's use meshes, clockwise mesh current in both meshes, $\mathbf{i}_1$ on the left, $\mathbf{i}_2$ on the right. $\omega = 4$, $\mathbf{Z}_1 = j(4\text{r/s})(2\text{H}) = j8\Omega$. $\mathbf{Z}_2 = j(4\text{r/s})(1\text{H}) = j4\Omega$.

$\mathbf{Z}_M = j(4\text{r/s})(1\text{H}) = j4\Omega$

KVL around left mesh

$-2\angle 0^\circ + 4\mathbf{i}_1 + j8\mathbf{i}_1 - j4\mathbf{i}_2 = 0$

$(4 + j8)\mathbf{i}_1 - j4\mathbf{i}_2 = 2 + j0$

KVL around right mesh

$j4\mathbf{i}_2 + 2\mathbf{i}_2 - j4\mathbf{i}_1 = 0$

$-j4\mathbf{i}_1 + (2 + j4)\mathbf{i}_2 = 0$

$$\begin{bmatrix} 4 + j8 & -j4 \\ -j4 & (2 + j4) \end{bmatrix} \begin{bmatrix} \mathbf{i}_1 \\ \mathbf{i}_2 \end{bmatrix} = \begin{bmatrix} 2 + j0 \\ 0 \end{bmatrix}$$

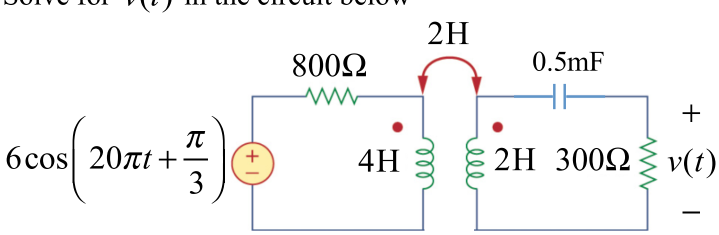
$$\begin{bmatrix} \mathbf{i}_1 \\ \mathbf{i}_2 \end{bmatrix} = \begin{bmatrix} 0.2059 + j\,0.1765 \\ 0.2353 - j\,0.0588 \end{bmatrix}$$

$v = 2\mathbf{i}_2 = 0.47 - j0.12 = 0.485\angle -14^\circ$

$v(t) = 0.481\cos(4t - 14^\circ)$

2:

Solve for $v(t)$ in the circuit below



$$\mathbf{V} = \mathbf{6}\angle\frac{\pi}{3} = 6\angle 60° = 3.0000 + j5.1962,\ \omega = 20\pi = 62.83\text{rad/sec}$$

$$\mathbf{Z}_N = j\omega 2\text{H} = j125.7\Omega$$

$$\mathbf{Z}_1 = j\omega 4\text{H} = j251.3\Omega$$

$$\mathbf{Z}_2 = j\omega 2\text{H} = j125.7\Omega$$

$$\mathbf{Z}_3 = \frac{1}{j\omega(0.5\times 10^{-3}\text{F})} = -j31.83\Omega$$

Define clockwise mesh currents

In left hand mesh

$$-6\angle 60° + 800\mathbf{I}_1 + j251.3\text{I}_1 - j125.7\mathbf{I}_2 = 0$$

$$(800 + j251.3)\mathbf{I}_1 - j125.7\mathbf{I}_2 = 6\angle 60°$$

In right hand mesh

$$-j31.8\mathbf{I}_2 + 300\mathbf{I}_2 + 125.7\mathbf{I}_2 - j125.7\mathbf{I}_1 = 0$$

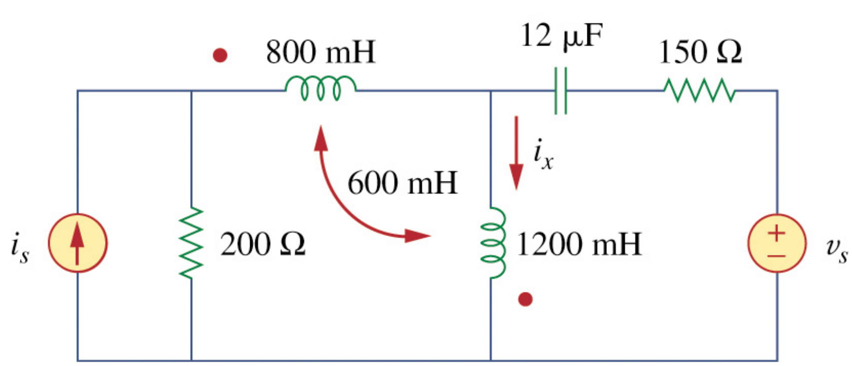
$$-j125.7\mathbf{I}_1 + (300 - j31.8 + j125.7)\mathbf{I}_2 = 0$$

$$\begin{bmatrix} (800 + j251.3) & -j125.7 \\ -j125.7 & (300 - j31.8 + j125.7) \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 6\angle 60° \\ 0 \end{bmatrix} = \begin{bmatrix} 3. + j5.2 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} (800 + j251.3) & -j125.7 \\ -j125.7 & (300 + j93.8) \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 6\angle 60° \\ 0 \end{bmatrix} = \begin{bmatrix} 3. + j5.2 \\ 0 \end{bmatrix}$$

$$\mathbf{v} = 300\mathbf{I}_2$$

3: Use mesh analysis to find i_x in the following circuit, when $i_s = 4\cos(600t)$, $v_s = 110\cos(600t + 30°)$ *Hint: Source transformation of the current source, after which the mesh current flows through both 800 mH and 1200 mH, inducing mutual inductance from each!*



Convert the inductors and capacitors to reactances.

$L_1 = 800\text{mH} = j600(0.8\text{H}) = j480\Omega$

$L_2 = 1200\text{mH} = j600(1.2\text{H}) = j720\Omega$

$M = 600\text{mH} = j600(0.6\text{H}) = j360\Omega$

$C = 12\mu\text{F} = \dfrac{1}{j600(1.2\times10^{-5}\text{F})} = -j139\Omega$

$\left(\text{note that } 600\text{mH} = \text{M} \le \sqrt{L_1 L_2} = 980\text{mH}\right)$

Transform i_s to $v_1 = i_s 200\Omega = 800\cos(600t)$

We are computing i_x, and have voltage sources, so mesh analysis is appropriate.

Assume i_1 in the left mesh, i_2 in the right, both flowing clockwise

The mutual inductance subtracts $j\omega M i_1$ twice in the i_1 mesh because i_1 flows into the L_1 dot and out of the L_2 dot. But we *add* the mutual inductance effect of i_2 because it flows into the L_2 dot, inducing a positive voltage at the L_1 dot. In the i_2 mesh there is only the self inductance of the i_1 current through the L_2 inductor and the mutual effect of i_1 in L_1 on L_2. i_1 flows into the L_1 dot, inducing a positive voltage at the L_2 dot.

Keeping these in mind, we can proceed by inspection

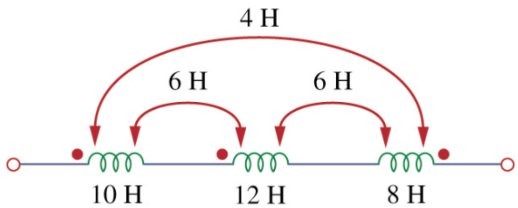
$$\begin{bmatrix} 200 + j480 + j720 - \underbrace{j360}_{\substack{L_1 \text{ effect}\\ \text{on } L_2\\ \text{due } i_1}} - j\,\underbrace{360}_{\substack{L_2 \text{ effect}\\ \text{on } L_1\text{due}\\ \text{to } i_1}} & -j720 + \underbrace{j360}_{\substack{L_2\text{effect}\\ \text{onf } L_1\\ \text{due to } i_2}} \\ -j720 + \underbrace{j360}_{\substack{L_1 \text{ effect}\\ \text{on } L_2\\ \text{due } i_1}} & j720 - j139 + 150 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = -\begin{bmatrix} -800\angle 0^\circ \\ 110\angle 30^\circ \end{bmatrix}$$

$$\begin{bmatrix} 200 + \text{j}480 & -j360 \\ -j360 & 150. + \text{j}581 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 800 \\ -952.6 - j55 \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 1.2961 \text{ - j}1.5643 \\ 0.8591 \text{ - j}0.5835 \end{bmatrix}$$

$i_x = i_1 - i_2 = \quad 0.4370 \text{ - j}0.9808 = 1.07\angle -66^\circ$

4: Determine the inductance of the three series-connected inductors shown below.



Look at the total voltage across the 3 inductors.

$$v = \underbrace{j\omega L_1 I + j\omega L_2 I + j\omega L_3 I}_{\text{the self-inductance of the three inductors due to the common current, } I} + \underbrace{\left(+j\omega M_{12} I - j\omega M_{13} I\right)}_{\text{the effect of current } I \text{ flowing though the first inductor on the voltages at the other two inducors, using the "dot" conventions}} + \underbrace{\left(+j\omega M_{12} I - j\omega M_{23} I\right)}_{\text{the effect of current } I \text{ flowing though the second inductor on the voltages at the other two inducors, using the "dot" conventions}} + \underbrace{\left(-j\omega M_{13} I - j\omega M_{23} I\right)}_{\text{the effect of current } I \text{ flowing though the third inductor on the voltages at the other two inducors, using the "dot" conventions}}$$

Factoring the common $j\omega$ factor and the common I factor from each term

$v = j\omega LI$, where $L = L_1 + L_2 + L_3 + 2M_{12} - 2M_{23} - 2M_{31}$

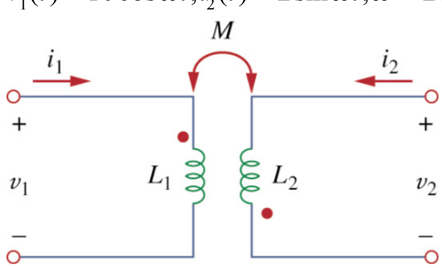
$= 10\text{H} + 12\text{H} + 8\text{H} + 2\times 6\text{H} - 2\times 6\text{H} - 2\times 4\text{H}$

$\boxed{L = 22\text{H}}$

5:

The coils shown below have $L_1 = 40\text{mH}, L_2 = 5\text{mH},$ and $k = 0.6$. (Pay attention to the defined direction of the currents!) Find $i_1(t), v_2(t)$ given that

$v_1(t) = 10\cos\omega t, i_2(t) = 2\sin\omega t, \omega = 2000$ rad/sec



$V_1 = i_1 \times j\omega L_1 - i_2 \times j\omega M$

$V_2 = i_2 \times j\omega L_2 - i_1 \times j\omega M$

$M = k\sqrt{L_1 L_2} = 0.6\sqrt{40\times10^{-3}\times5\times10^{-3}} = 8.5\text{mH}$

From V_1 equation above: $\dfrac{V_1 + i_2 \times j\omega M}{j\omega L_1} = i_1$

$$i_1 = \frac{10 + (-j2)\left(j2000 \text{ r/s} \times 8.5\times10^{-3}\Omega\text{-sec}\right)}{j\left(2000 \text{ r/s} \times 40\times10^{-3}\Omega\text{-sec}\right)}$$

$= -j0.55 = 0.55\angle -90^\circ$

$\boxed{i_1(t) = 0.55\sin 2000t}$

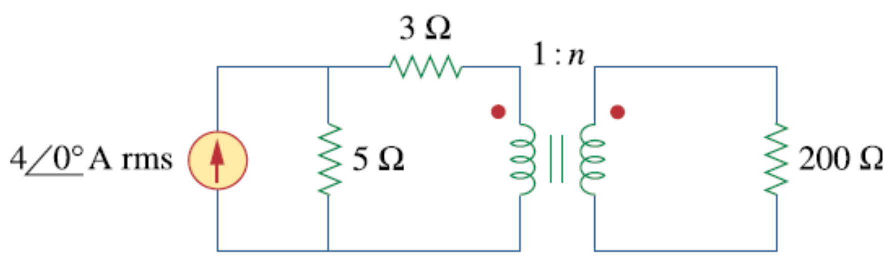
From V_2 equation,

$V_2 = (-j2\text{A})\times(j2000 \text{ r/s}\times5\times10^{-3}\Omega\text{-sec}) - (-j0.55\text{A})\times(j2000\text{r/s}\times8.5\times10^{-3}\ \Omega\text{-s})$

$10.65 + j0$

$\boxed{V_2(t) = 10.65\cos 2000t}$

6: Find n for the maximum power supplied to the 200 ohm load. Then determine the power to the 200 ohm load if *n*=10.



Transform the current source to get
$v_s = 4 \times 5\Omega = 20\angle 0°$ V.

Reflect the 200Ω to the primary side: $Z_{EQ} = \frac{Z}{n^2} = \frac{200\Omega}{n^2}$

With the voltage source transformation we have a voltage source and 5+3Ω in series. So the voltage source and series resistance represents the Thevenin equivalent of the left (primary) side.

For maximum power transfer, $\mathbf{Z}_L = \mathbf{Z}_{TH}^* = 8\Omega = \frac{200\Omega}{n^2}$

$n^2 = \frac{200}{8} = 25,\ n = 5.$

If $n = 10$ then the reflected impedence is $\frac{200}{10^2} = 2\Omega.$

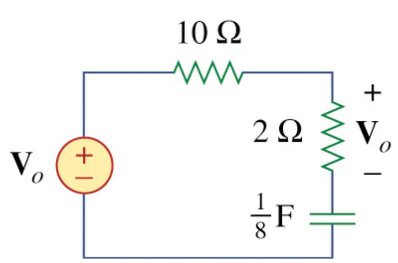
The primary current is $\frac{20\angle 0°}{8\Omega + 2\Omega} = 2\angle 0°\text{A}.$

The voltage across the load is $20\angle 0° \times \frac{2\Omega}{8\Omega + 2\Omega} = 4\angle 0°\text{V}$

$P = \left|\mathbf{V}_{RMS}\mathbf{I}_{RMW}{}^*\right|\cos(\theta_v - \theta_i) = 8\cos(0) = 8\text{W}$

7:

Obtain the transfer function $\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i}$ of the following circuit. **Note** that there is a "typo" in the circuit figure, and the voltage source should be labeled $\mathbf{V}_i$.



$\mathbf{H}(\omega) = \frac{\mathbf{V}_0}{\mathbf{V}_i}$. This is a series RLC circuit, and therefore a voltage divider.

$$\mathbf{H}(\omega) = \frac{2\Omega}{10 + 2 + \frac{1}{j\omega(1/8)}} = \frac{j2\omega/8}{1 + j10\omega/8} = \frac{j\omega/4}{1 + j\omega/0.8}$$