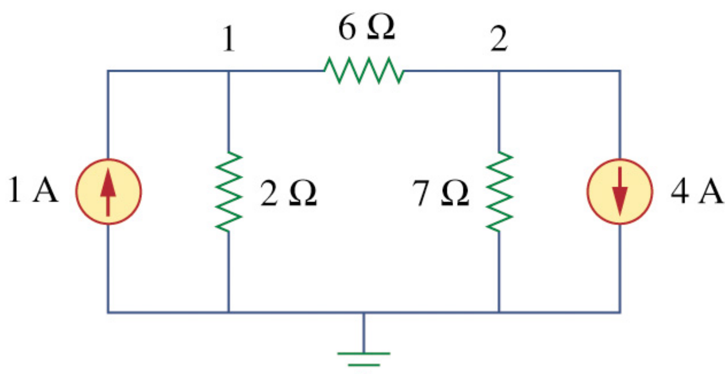
CMPE306 Fall 2019 Homework L05
Assigned 11/25/19 Due 12/2/19

1.
Obtain the node voltages in this circuit using nodal analysis by inspection. Remember that the coefficients in the matrix are *conductances*, not *resistances.* Use MATLAB to solve for the node voltage values.



$$\begin{bmatrix} \frac{1}{2}+\frac{1}{6} & -\frac{1}{6} \\ -\frac{1}{6} & \frac{1}{6}+\frac{1}{7} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$$

```
>> A=[1/2+1/6 -1/6;-1/6 1/6+1/7];
>> b=[1;-4];
>> vv=A\b; % "\" "divides" by A on the left, that is A^(-1)b
>> vv
vv =
   -2
  -14
```

2: Use nodal analysis by inspection and then find V_0 in the following circuit (updated 2/9/2017).

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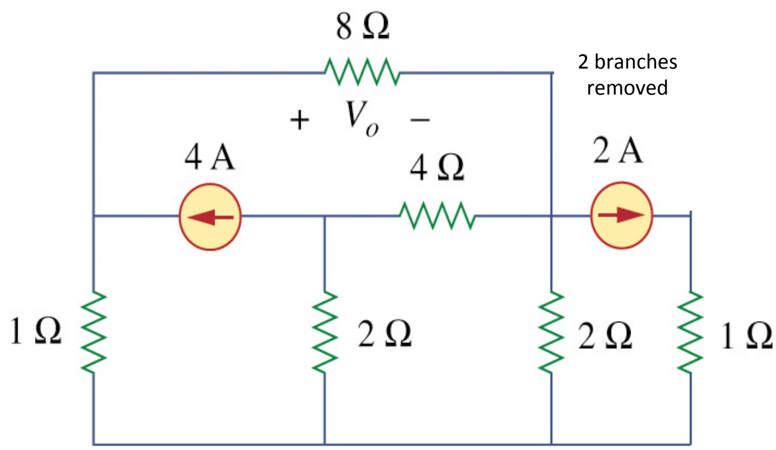

$V_0 = v_1 - v_2$

$$\begin{bmatrix} \frac{1}{1}+\frac{1}{8} & 0 & -\frac{1}{8} & 0 \\ 0 & \frac{1}{2}+\frac{1}{4} & -\frac{1}{4} & 0 \\ -\frac{1}{8} & -\frac{1}{4} & \frac{1}{4}+\frac{1}{2}+\frac{1}{8} & 0 \\ 0 & 0 & 0 & \frac{1}{1} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 4 \\ -4 \\ -2 \\ 2 \end{bmatrix}$$

```
>> A=[1/1+1/8, 0, -1/8, 0;
    0, 1/2+1/4, -1/4, 0;
  -1/8, -1/4, 1/4+1/2+1/8, 0;
    0,   0,    0, 1/1]
A =
   1.1250         0   -0.1250         0
        0    0.7500   -0.2500         0
  -0.1250   -0.2500    0.8750         0
        0         0         0    1.0000
>> b=[4;-4;-2;2];
>> vv=A\b
vv =
   3.1429
  -6.5714
  -3.7143
   2.0000
```

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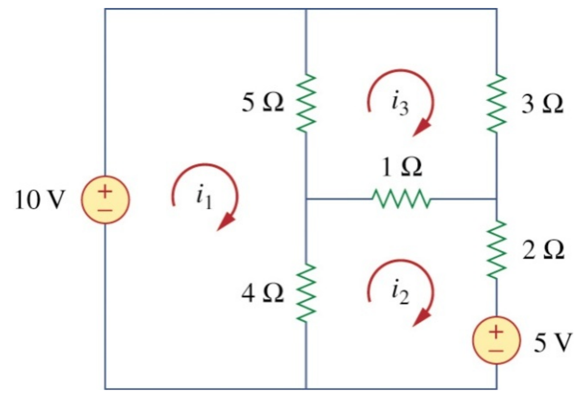
3.
Obtain the matrix equation for the mesh currents in the circuit at the right by inspection, then solve for the mesh currents using MATLAB.



$$\begin{bmatrix} 5+4 & -4 & -5 \\ -4 & 4+1+2 & -1 \\ -5 & -1 & 5+1+3 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = -\begin{bmatrix} -10 \\ 5 \\ 0 \end{bmatrix} = \begin{bmatrix} 10 \\ -5 \\ 0 \end{bmatrix}$$

```
>> A=[5+4   -4   -5;
     -4  4+1+2   -1;
     -5   -1   5+1+3]
A =
    9   -4   -5
   -4    7   -1
   -5   -1    9
>> b=-[-10;5;0]
b =
   10
   -5
    0
>> ii=A\b
ii =
   2.0854
   0.6533
   1.2312
```

4. Determine the center frequency and the bandwidth of the frequency selective circuits in the following figure. You may use inspection techniques if your wish. Treat $\mathbf{H}(\omega) = \dfrac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}$ and apply the condition for resonance.

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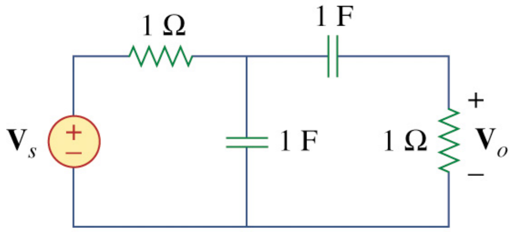
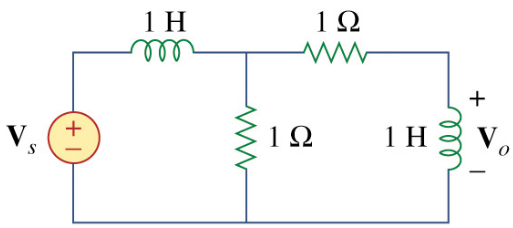


(a)



(b)

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I apologize. This, again, is a not-completely-defined problem from the textbook.

Let $\mathbf{H}(\omega)=\dfrac{\mathbf{V}_O(\omega)}{\mathbf{V}_S(\omega)}$ (this is the part that was missing).

Find the center frequency and bandwidth of the circuit in (a)

Define $\mathbf{I}$ as the current flowing out of the source $\mathbf{V}_S$. By Ohm's Law, $\mathbf{I}=\dfrac{\mathbf{V}_S}{\mathbf{Z}_T}$,

where $\mathbf{Z}_T$ is total impedance of the circuit. $\mathbf{Z}_T = R+\dfrac{1}{j\omega C}\,||\left(R+\dfrac{1}{j\omega C}\right)$, where $R=1\Omega$ and $C=1\text{F}$

Switch to s for ease of notation, $\mathbf{Z}_T = R+\dfrac{1}{sC}\,||\left(R+\dfrac{1}{sC}\right)=R+\dfrac{1}{sC}\,||\left(\dfrac{1+sRC}{sC}\right)$

$$=R+\frac{\frac{1}{sC}\left(\frac{1+sRC}{sC}\right)}{\frac{1}{sC}|+\left(\frac{1+sRC}{sC}\right)}=R+\frac{\frac{1}{s^2C^2}(1+sRC)}{\left(\frac{2+sRC}{sC}\right)}=R+\frac{(1+sRC)}{sC(2+sRC)}$$

$$=\frac{RsC(2+sRC)+(1+sRC)}{sC(2+sRC)}=\frac{(2sRC+s^2R^2C^2)+(1+sRC)}{sC(2+sRC)}=\frac{(1+3sRC+s^2R^2C^2)}{sC(2+sRC)}$$

$\mathbf{I}=\dfrac{\mathbf{V}_S}{\mathbf{Z}_T}=\dfrac{\mathbf{V}_S sC(2+sRC)}{(1+3sRC+s^2R^2C^2)}$ Don't worry about the math yet!

$\mathbf{V}_O=\mathbf{I}_1\times R$, where $\mathbf{I}_1$ is the current in the $R+\dfrac{1}{sC}$ branch. This is a current divider,

so $\mathbf{I}_1=\dfrac{\frac{1}{sC}}{\frac{1}{sC}+R+\frac{1}{sC}}\mathbf{I}=\dfrac{\mathbf{I}}{2+sRC}=\dfrac{1}{2+sRC}\times\dfrac{\mathbf{V}_S sC(2+sRC)}{(1+3sRC+s^2R^2C^2)}$

$\mathbf{V}_O=\mathbf{I}_1 R=\dfrac{\mathbf{V}_S sRC}{(1+3sRC+s^2R^2C^2)}$, so $\mathbf{H}(s)=\dfrac{\mathbf{V}_O}{\mathbf{V}_S}=\dfrac{sRC}{(1+3sRC+s^2R^2C^2)}$

Just checking that I did everything right, s has units of $\sec^{-1}$, RC is sec, so we

have $\mathbf{H}(\omega)=\dfrac{\sec^{-1}\sec}{(1+\sec^{-1}\sec+\sec^{-2}\sec^2)}=\dfrac{1}{1}$ which is OK for $\dfrac{\text{volts}}{\text{volt}}$

Look at the denominator, and rearrange in our standard form as a monic polynomial with 1 as the coefficient of the highest power of s:

$$\mathbf{H}(s)=\frac{sRC}{(R^2C^2)\left(s^2+\frac{3sRC}{R^2C^2}+\frac{1}{R^2C^2}\right)}=\frac{s/(RC)}{\left(s^2+\frac{3s}{RC}+\frac{1}{R^2C^2}\right)}$$ with units of $\dfrac{\sec^{-2}}{\sec^{-2}}$.

From our standard form, $\alpha=\dfrac{3}{2RC}$ $\omega_0=\dfrac{1}{RC}$.

So the center frequency is $\omega_0=\dfrac{1}{1\Omega\times 1\text{F}}=1$ r/s. $B=2\alpha=\dfrac{2\times 3}{2RC}=\dfrac{3}{1\Omega\times 1\text{F}}=3$ r/s.

the process is the same for this problem

$$\mathbf{Z}_T = sL + \left(R \parallel R + sL\right) = sL + \frac{R(1+sL)}{R+R+sL} = sL + \frac{(R+sL)}{\left(2+s\frac{L}{R}\right)} = \frac{2sL + s^2\frac{L^2}{R} + R + sL}{\left(2+s\frac{L}{R}\right)}$$

$$= \frac{R\left(s^2\frac{L^2}{R^2} + 3s\frac{L}{R} + 1\right)}{\left(2+s\frac{L}{R}\right)} \text{ we'll simplify later.}$$

$$\mathbf{I} = \frac{\mathbf{V}_S\left(2+s\frac{L}{R}\right)}{R\left(s^2\frac{L^2}{R^2} + 3s\frac{L}{R} + 1\right)},\ \mathbf{I}_1 = \mathbf{I} \times \frac{R}{\left(2R+sL\right)} = \mathbf{I} \times \frac{1}{\left(2+s\frac{L}{R}\right)}$$

$$= \frac{\mathbf{V}_S\left(2+s\frac{L}{R}\right)}{R\left(s^2\frac{L^2}{R^2} + 3s\frac{L}{R} + 1\right)} \times \frac{1}{\left(2+s\frac{L}{R}\right)} = \frac{\mathbf{V}_S}{R\left(s^2\frac{L^2}{R^2} + 3s\frac{L}{R} + 1\right)}$$

$$\mathbf{V}_O = sL\mathbf{I} = \frac{\mathbf{V}_S sL}{R\left(s^2\frac{L^2}{R^2} + 3s\frac{L}{R} + 1\right)}$$

$$\mathbf{H}(s) = \frac{\mathbf{V}_O}{\mathbf{V}_S} = \frac{sL}{R\left(s^2\frac{L^2}{R^2} + 3s\frac{L}{R} + 1\right)} = \frac{s(L/R)}{\left(s^2\frac{L^2}{R^2} + 3s\frac{L}{R} + 1\right)} = \frac{s(L/R)}{\frac{L^2}{R^2}\left(s^2 + 3s\frac{R}{L} + \frac{R^2}{L^2}\right)}$$

$$\mathbf{H}(s) = \frac{s(R/L)}{\left(s^2 + 3s\frac{R}{L} + \frac{R^2}{L^2}\right)}.$$

Following the same process $\omega_0 = \frac{R}{L} = 1$ r/s. $\alpha = \frac{3R}{2L} = \frac{3}{2}, B = 2\alpha = 3$ r/s.

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Both problems seem WRONG, because $\omega_0 \pm \frac{BW}{2}$ gives the half power (-3 dB) band edges as $\omega_1 = -0.5$, and we don't really know what a negative frequency means in this context.

BUT, the $\omega_0 \pm \frac{BW}{2}$ approximation only holds when BW $\ll \omega_0$, which isn't the case here, so we have to back to the book (or lecture).

$\omega_0 = \sqrt{\omega_1 \omega_2}$, the *geometric* mean, instead of the *arithmetic* mean.

BW=$\omega_2 - \omega_1 = 3$ rps, so $\omega_2 = 3 + \omega_1$. Substituting in the equation

for $\omega_0 = \sqrt{\omega_1(3+\omega_1)}$, or, $\omega_0^2 = 3\omega_1 + \omega_1^2$. Substituting the value of

$\omega_0 = 1$, $\omega_1^2 + 3\omega_1 - 1 = 0$, $\omega_1 = \frac{-3}{2} \pm \frac{\sqrt{9-4(-1)}}{2} = \frac{-3}{2} \pm \frac{\sqrt{13}}{2}$.

Discarding the negative root, because $\omega_1 > 0$, $\omega_1 = \frac{-3}{2} + \frac{\sqrt{13}}{2} = 0.3028$

$\omega_2 = 3 + 0.3028 = 3.3028$.

Just checking $\omega_0 = 1 = \sqrt{0.3028(3.3028)} = 1!!$