1:

If $v(t)=160\cos 50t$ V and $i(t)=-20\sin(50t-30^{\circ})$, calculate the instantaneous power, average power, and apparent power. The apparent power, S, is the magnitude of the complex power,

$\mathbf{S}=\frac{1}{2}\mathbf{VI}^{*}$,

Taking $v(t)$ and $i(t)$ to be peak voltage and current, $\mathbf{S}=\frac{1}{2}\mathbf{VI}^{*}$.

$$\mathbf{V}=160\cos 50t=160\angle 0^{\circ},$$

$$\mathbf{I}=-20\sin(50t-30^{\circ})=20\cos(50t-30^{\circ}-90^{\circ}+180^{\circ})=20\cos(50t+60^{\circ})=20\angle 60^{\circ}$$

$$\mathbf{VI}^{*}=(160)(20)\angle(0-60^{\circ})=3200\angle -60^{\circ}$$

$$\mathbf{S}=\frac{1}{2}\mathbf{VI}^{*}=1600\angle -60^{\circ}$$

$$S=\left|\mathbf{S}\right|=1600, P=S\cos(\theta_V-\theta_I)=S\cos(-60^{\circ})=800 \text{ watts}$$

2:

An ac motor with impedance $\mathbf{Z}_L=4.2+j3.6\Omega$ is supplied by a 220V, 60 Hz source. Find power factor (pf), P, and Q.

Assume 220V is rms voltage, and take the peak of this cosine wave to be the reference phase.

$$\mathbf{S}=\mathbf{V}_{RMS}\mathbf{I}^{*}_{RMS}=\frac{\mathbf{V}_{RMS}\mathbf{V}^{*}_{RMS}}{\mathbf{Z}^{*}}=\frac{\left|\mathbf{V}_{RMS}\right|^{2}}{\mathbf{Z}^{*}}=\frac{220^{2}\text{V}^{2}}{4.2-j36}=\frac{48400\text{V}^{2}}{5.53\angle +40.6^{\circ}}=8749.5\angle -40.6^{\circ}$$

$$P=\left|\mathbf{S}\right|\cos(-40.6^{\circ})=6643\text{W, pf}=\cos(-40.6^{\circ})=0.759=\frac{P}{\left|\mathbf{S}\right|}$$

$$Q=\left|\mathbf{S}\right|\sin(-40.6^{\circ})=-5694\text{VAR}$$

If we assume that 220V is the peak voltage, we have to divide by 2

$P=3321.5\text{W}, Q=-2847\text{VAR}$

We could report Q as 2847VAR leading instead of -2847VAR.

3:

For the following voltage and current phasors, calculate the complex power, apparent power, real power, and reactive power. Specify whether the pf is leading or lagging.

7.1 $\mathbf{S} = \mathbf{VI}^* = \left(220\angle 30^\circ\right)\left(0.5\angle -60^\circ\right) = 110\angle -30^\circ = 95.2 - j55 \text{ VA}$

$S = |\mathbf{S}| = 110,\ P = 95.2\text{W},\ Q = 55$ VAR leading (because $\theta_V - \theta_I = -30 < 0$)

7.2 $\mathbf{S} = \mathbf{VI}^* = \left(250\angle -10^\circ\right)\left(6.2\angle +25^\circ\right) = 1550\angle +15^\circ = 1497 + j401 \text{ VA}$

$S = |\mathbf{S}| = 1550\text{VA},\ P = 1497\text{W}, Q = 401$ VAR lagging

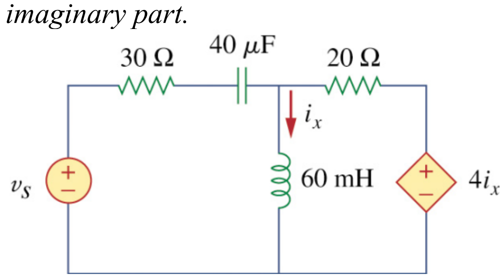
7.3 $\mathbf{S} = \mathbf{VI}^* = \left(120\angle 0\right)\left(2.4\angle +15\right) = 288\angle +15^\circ = 278 + j74.5 \text{ VA}$

$S = |\mathbf{S}| = 288$ VA, $P = 278\text{W},\ Q = 74.5\text{VAR}$ lagging

4:

Find the complex power delivered by v_S to the network below. Let $v_s(t) = 100\cos 2000t$ V.

Hint: AC dependent sources are treated the same way they are treated in DC circuits, except with complex arithmetic. So $4i_x$ will, in general, have a magnitude and phase or a real and imaginary part.



$v_s(t) = 100\cos\left(2000t\right) = 100\angle 0^\circ$. We will assume that this is peak volts

$\omega = 2000$ rad/s;

$$40\mu F \rightarrow \frac{1}{j(2\times 10^3)(40\times 10^{-6})} = -j12.5\Omega$$

$$60\text{mH} \rightarrow j(2\times 10^3)(60\times 10^{-3}) = j120\Omega$$

Use mesh with i_1 in left mest, i_2 in right mesh, both clockwise.

$\omega = 2000$r/s

Mesh 1

$$-v_S + i_1\left(30 + \frac{1}{j2000(4\times 10^{-4})} + j2000\times 6\times 10^{-2}\right) - i_2\left(j2000\times 6\times 10^{-2}\right) = 0$$

$$i_1\left(30 - j12.5 + j120\right) - i_2(j120) = 100$$

Mesh 2

$$i_2(20 + j2000\times 6\times 10^{-2}) - i_1(j2000\times 6\times 10^{-2}) + 4\mathbf{i}_x = 0$$

$$-i_1\left(j120\right) + i_2(20 + j120) = -4\text{i}_x \text{, but } \mathbf{i}_x = i_1 - i_2 =$$

$$-i_1(j120 - 4) + i_2(20 - 4 + j120) = 0$$

$$i_1(4 - j120) + i_2(16 + j120) = 0$$

Or meshes by inspection

$$\begin{bmatrix} 30 - j12.5 + j120 & -j120 \\ -j120 & 20 + j120 \end{bmatrix}\begin{bmatrix} \mathbf{i}_1 \\ \mathbf{i}_2 \end{bmatrix} = -\begin{bmatrix} -100 \\ 4\mathbf{i}_x \end{bmatrix} = \begin{bmatrix} 100 \\ -4\mathbf{i}_x \end{bmatrix}$$

$$\mathbf{i}_x = \mathbf{i}_1 - \mathbf{i}_2$$

$$\begin{bmatrix} 30 - j12.5 + j120 & -j120 \\ -j120 & 20 + j120 \end{bmatrix}\begin{bmatrix} \mathbf{i}_1 \\ \mathbf{i}_2 \end{bmatrix} = \begin{bmatrix} 100 \\ -4\left(\mathbf{i}_1 - \mathbf{i}_2\right) \end{bmatrix}$$

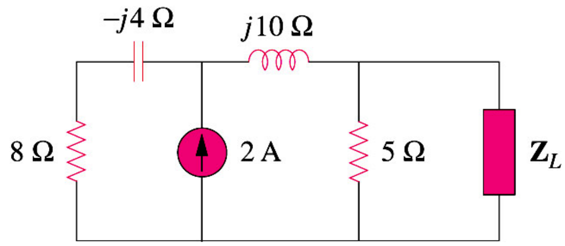
$$\begin{bmatrix} 30 - j12.5 + j120 & -j120 \\ 4 - j120 & -4 + 20 + j120 \end{bmatrix}\begin{bmatrix} \mathbf{i}_1 \\ \mathbf{i}_2 \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{i}_1 \\ \mathbf{i}_2 \end{bmatrix} = \begin{bmatrix} 1.9374 + j0.3855 \\ 1.8319 + j0.6943 \end{bmatrix} = \begin{bmatrix} 1.9753\angle 11.3^\circ \\ 1.9591\angle 20.8^\circ \end{bmatrix}$$

$$\mathbf{S} = \frac{1}{2}\mathbf{VI}^* = \frac{1}{2}\times\left(100\angle 0\right)\left(1.975\angle -11.3^\circ\right) = 197.5\angle -11.3^\circ = 193.5 - j38.6 \text{ VA}$$

5:

Find the load impedance that absorbs the maximum power. Find the maximum power in the load impedance. *Hint: Think Thevenin!*



This is the same circuit as Problem 1 and 2 in HW L03. So we already know that

$$\mathbf{Z}_{TH} = 3.4146 + j0.7317 = 3.4922\angle 12.1^\circ$$

The load impedance for maximum power transfer is

$$\mathbf{Z}_L = \mathbf{Z}_{TH}^* = 3.4146 - j0.7317 = 3.4922\angle -12.1^\circ$$

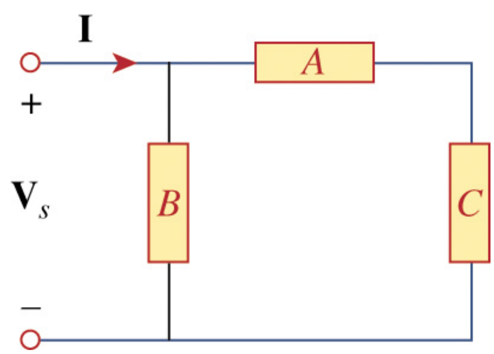
The power in that load is (see lecture slide)

$$P_{MAX} = \frac{|\mathbf{V}_{TH}|^2}{8R_{TH}} = \frac{6.247^2}{8(3.9)} = 1.43\text{W}$$

6:

In the circuit show below, device A receives 2 kW at 0.8 pf lagging, device B receives 3kVA at 0.4 pf leading, and device C is inductive (i.e. “leading”) and consumes 1 kW with a reactive component of 500 VAR.

(a) Determine the power factor (pf) of the entire system.
(b) Find **I** given $\mathbf{V}_S = 120\angle 45^\circ$ rms



Element A: $P_A = 2\text{kW}$@power factor of 0.8 lagging.

$$\text{p.f.} = \cos(\theta_v - \theta_i) = 0.8, (\theta_v - \theta_i) = \cos^{-1}(0.8) = 36.9^\circ$$

"*lagging*" > 0, so this is +36.9°.

$$\text{p.f.} = \frac{P_A}{|\mathbf{S}_A|} = 0.8, \text{ so } |\mathbf{S}_A| = \frac{P_A}{0.8} = 2.5\text{kVA}$$

$$\mathbf{S}_A = |\mathbf{S}_A| e^{j(\theta_v - \theta_i)} = 2000 + j1500 \text{ VA}$$

Element B: 3kVA@p.f = 0.4, leading

$S_B = |\mathbf{S}_B| = 3\text{kVA},\ (\theta_v - \theta_i) = \cos^{-1}(0.4) = 66.42^\circ, \text{"}leading\text{"} < 0,$

$\mathbf{S}_B = 3\text{kVA}\left(\cos(66.42^\circ) - j\sin(66.42^\circ)\right) = 1200 + j2750 \text{ VA}$

Element C: is inductive $\Rightarrow$ leading p.f. $\Rightarrow \theta_v - \theta_i > 0$

$P = 1000\text{W},\ Q = 500\text{VAR}$

$\mathbf{S}_C = 1000 + j500 \text{ VA}$

$\mathbf{S}_{SOURCE} = -\left(\mathbf{S}_A + \mathbf{S}_B + \mathbf{S}_C\right) = -\left[(2000 + 1200 + 1000) + j(1500 - 2750 + 500)\right]$

$= -4200 + j750 = 4266\angle 169.9$ VA, where the minus sign is from the passive sign convention.

$\mathbf{S}_{SOURCE} + \left(\mathbf{S}_A + \mathbf{S}_B + \mathbf{S}_C\right) = 0,$ as required by Conservation of Energy.

$$\text{p.f.(system)} = \frac{\text{Re}\left(\mathbf{S}_A + \mathbf{S}_B + \mathbf{S}_C\right)}{\left|\left(\mathbf{S}_A + \mathbf{S}_B + \mathbf{S}_C\right)\right|} = \frac{-P_S}{|\mathbf{S}_S|} = 0.9844$$

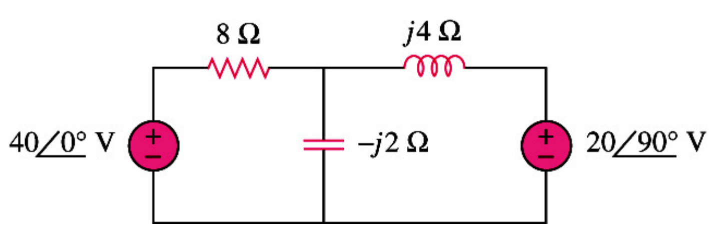
Assuming that the voltage is given as rms, $\mathbf{S}_S = -\mathbf{V}_S\mathbf{I}^*$ (passive sign)

$$\mathbf{I} = \left(\frac{-\mathbf{S}_S}{\mathbf{V}_S}\right)^* = \left(\frac{4266\angle(169.9 - 180)^\circ}{120\angle 45^\circ}\right)^* = 35.55\angle -\left(-55^\circ\right)$$

$= 35.55\angle 55^\circ = 20.3 + j29.2$ Arms

7:

Find the complex power associated with each of the five elements in this circuit, then determine the real power in each case. The voltages are given in RMS units (Volts rms)



We don't need to convert to impedances, as that has already been done.

Using meshes

Left hand mesh:

$$-40\angle 0 + \mathbf{i}_1 \times 8\Omega + (\mathbf{i}_1 - \mathbf{i}_2)(-j2\Omega) = 0$$

$$\left(8 - j2\right)\mathbf{i}_1 - (-j2)\mathbf{i}_2 = 40 + j0$$

$$\left(8 - j2\right)\mathbf{i}_1 + (j2)\mathbf{i}_2 = 40 + j0$$

Right hand mesh

$$\mathbf{i}_2 \times (j4) + 20\angle 90^\circ + (\mathbf{i}_2 - \mathbf{i}_1)(-j2) = 0$$

$$-(-j2)\mathbf{i}_1 + (j4 - j2)\mathbf{i}_2 = -20\angle 90^\circ = 20\angle -90^\circ = -j20$$

$$(j2)\mathbf{i}_1 + (j2)\mathbf{i}_2 = -20\angle 90^\circ = 20\angle -90^\circ = -j20$$

$$\begin{bmatrix} 8 - j2 & j2 \\ j2 & -j2 \end{bmatrix} \begin{bmatrix} \mathbf{i}_1 \\ \mathbf{i}_2 \end{bmatrix} = \begin{bmatrix} 40 \\ -j20 \end{bmatrix}$$

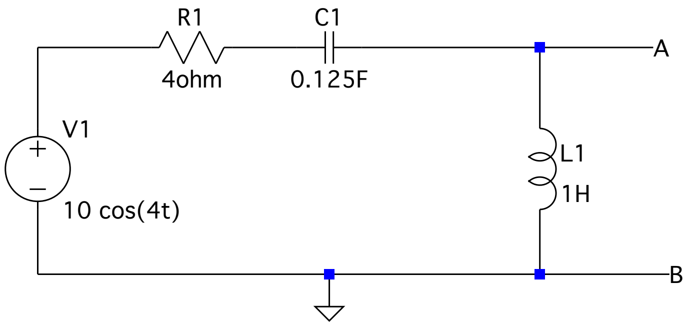
(note this is the same matrix equation we get with inspection using **Z**, not *R*. Try it!)

MATLAB

$$\begin{bmatrix} \mathbf{i}_1 \\ \mathbf{i}_2 \end{bmatrix} = \begin{bmatrix} 5.0000\ -j2.5000\text{i} \\ 15.0000\ -j2.5000\text{i} \end{bmatrix} = \begin{bmatrix} 5.5902\angle -26.6^\circ \\ 15.2069\angle -9.4^\circ \end{bmatrix}$$

The voltage across the capacitor is $(\mathbf{i}_1 - \mathbf{i}_2) \times (-j2) = j20\text{V} = 20\angle 90^\circ\,\text{V}$

8. Find the (possibly complex) load impedance connected across terminals A-B that absorbs the maximum power. Find the maximum power when matched load is present. The sinusoidal amplitude is the peak voltage, not the rms.



Maximum power problems are *always* Thevenin problems.

This is already an open circuit. Turn off the independent source, making it a wire. Then $\mathbf{Z}_{TH} = j\omega L \| (R + \frac{1}{j\omega C})$

$$= \frac{j(4\text{r/s})(1\text{H}) \times (4\Omega + \frac{1}{j(4\text{r/s})(0.125\text{F})})}{j(4\text{r/s})(1\text{H}) + (4\Omega + \frac{1}{j(4\text{r/s})(0.125\text{F})})} = \frac{j4\Omega(4 - j2\Omega)}{j4\Omega + 4 - j2\Omega} = 3.2 + j2.4\Omega$$

To find $\mathbf{V}_{TH}$, we have a voltage divider

$$\mathbf{V}_{TH} = \mathbf{V}_S \times \frac{j4\Omega}{4 + j2\Omega} = 10 \times (0.4 + j0.8) = 4 + j8\text{V} = 1.1\angle 63.4^\circ$$

$$\mathbf{Z}_L = \mathbf{Z}_{TH}^* = 3.2 - j2.4\Omega$$

$$P = \frac{|\mathbf{V}_{TH}|^2}{8R_{TH}} = \frac{1.21\text{V}^2}{8 \times 3.2\Omega} = 0.0313\text{W=}31.3\text{mW}$$