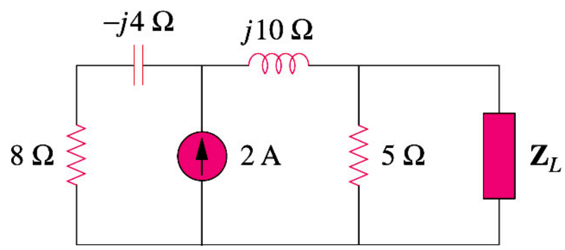
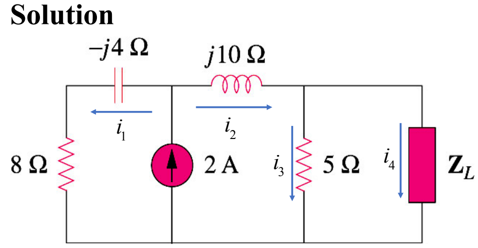
Content from this homework may appear on the second exam, scheduled for November 13, 2019. We will review this homework in class on Monday November 11, so make sure you do it before then. Use this as your opportunity to study this new material.

1:

If the load resistance is 4Ω, and the load reactance is $-j2\Omega$, find the node voltages. Use the complex form of Ohm's Law, $\mathbf{V} = \mathbf{IZ}$, where $\mathbf{V}$, $\mathbf{Z}$ and $\mathbf{I}$ are complex values. Nodal analysis and mesh analysis work just fine, but are done with complex arithmetic! Because the complex impedances are given, you may assume that the source is $2\angle 0^{\circ}\,\text{A} = 2\cos(\omega t)\,\text{A}$.



Solution



$\mathbf{Z}_L = 4 - j2\Omega$

Nodal analysis. Defining the branch currents as shown

Node 1 (on the left)

$2 - i_2 - i_1 = 0$

$2 = i_1 + i_2$

Node 2

$i_2 - i_3 - i_4 = 0, \text{ or } i_3 + i_4 - i_2 = 0$

Dictionary

$$i_1 = \frac{v_1}{8-j4}, i_2 = \frac{v_1 - v_2}{j10}, i_3 = \frac{v_2}{5}, i_4 = \frac{v_2}{4-j2}$$

Substitution

$$\frac{v_1}{8-j4} + \frac{v_1 - v_2}{j10} = 2, \text{ simplifying } \left(\frac{1}{8-j4} + \frac{1}{j10}\right)v_1 - \left(\frac{1}{j10}\right) = 2$$

$$\frac{v_2}{5} + \frac{v_2}{4-j2} - \frac{v_1 - v_2}{j10} = 0, \text{ simplifying } -\left(\frac{1}{j10}\right)v_1 + \left(\frac{1}{5} + \frac{1}{4-j2} + \frac{1}{j10}\right)v_2 = 0$$

Or use nodes by inspection to get the same result.

$$\begin{bmatrix} \frac{1}{8-j4} + \frac{1}{j10} & -\frac{1}{j10} \\ -\frac{1}{j10} & \frac{1}{j10} + \frac{1}{5} + \frac{1}{4-j2} \end{bmatrix} \begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \end{bmatrix} = \begin{bmatrix} 2 + j0 \\ 0 \end{bmatrix}$$

MATLAB

$$\begin{bmatrix} 0.1000+j\,0.0500 & 0.0000 + j0.1000 \\ 0.0000 + j\,0.1000 & 0.4000 + j0.0000 \end{bmatrix} \begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \end{bmatrix} = \begin{bmatrix} 13.7931 + j5.5172 \\ 1.3793 - j3.4483 \end{bmatrix} = \begin{bmatrix} 14.8556\angle 21.8^\circ \\ 3.7139\angle -68.2^\circ \end{bmatrix}$$

2: Repeat problem 4 by treating $\mathbf{Z}_L$ as the load and using Thevenin's Theorem. You will find the Thevenin impedance and the (sinusoidal) Thevenin voltage. *Hint: Do not panic! Just do what you would do with resistors using the complex impedance of the elements.*

Thevenin works fine with AC circuits.

To find $\mathbf{Z}_{TH}$, turn off the independent sources. In this case, that's 2A.

A circuit source with 0 A is equivalent to an open circuit.

The load then sees $5\Omega \| (8 - j4 + j10) = \dfrac{5(8 - j4 + j10)}{5 + 8 + j6} =$

$\dfrac{40 - j30}{13 + j6} =$ 3.4146 + 0.7317i $= 3.4922\angle 12.1^{\circ}$

For $\mathbf{V}_{TH}$ turn the source back on. With $\mathbf{Z}_L$ removed (Thevenin is open circuit), we have 2A in parallel with $(8 - j4) \| (5 + j10)$. Parallel impedances are a current divider.

In this situation, $i_2 = i_3$, and $i_4 = 0$. So

$i_2 = \dfrac{8 - j4}{13 + j10} \times 2\text{A} = 0.7805 - 0.9756\text{i} = 1.2494\angle -51.3^{\circ}$

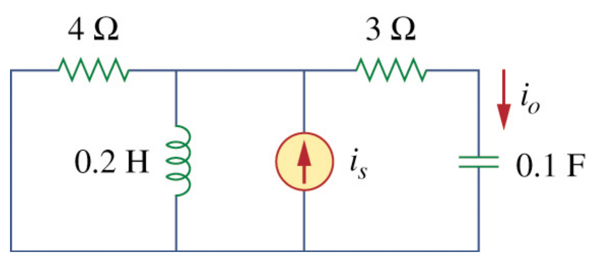
$\mathbf{V}_{TH} = 5\Omega \times i_2 = 3.9024 - 4.8780\text{i} = 6.247\angle -51.3^{\circ}$

Replacing the load gives a voltage divider

$\mathbf{V}_L = \mathbf{V}_{TH} \times \dfrac{\mathbf{Z}_L}{\mathbf{Z}_L + \mathbf{Z}_{TH}} = 6.247\angle -51.3^{\circ} \times \dfrac{4 - j2}{4 - j2 + 3.4 - j0.73}$

$= 1.3793 - 3.4483\text{i} = 3.7\angle -68.2$ as in Problem 1, $\mathbf{V}_2$

3: If $i_S(t) = 5\cos(2\pi ft)$ and $f = 5\text{Hz}$, write the phasor and rectangular forms for $i_0(t)$.



$\omega = 2\pi f = 10\pi$

$\mathbf{Z}_L = j\omega L = j(0.2)(10\pi) = j2\pi\Omega$

$\mathbf{Z}_C = \dfrac{1}{j\omega C} = \dfrac{1}{j(0.1)(10\pi)} = -j\dfrac{1}{\pi}\Omega$

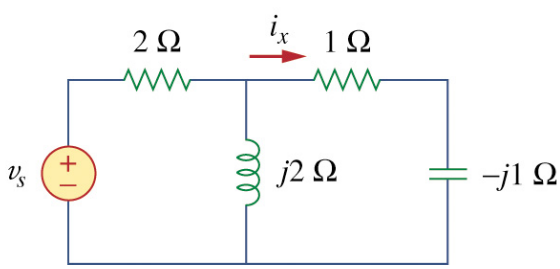
This is a nodal analysis problem with one node!

$$\left(\frac{1}{4} + \frac{1}{j2\pi} + \frac{1}{3 - j\frac{1}{\pi}}\right) v = i_s = 5\angle 0^\circ$$

$$v = \frac{5}{\left(\frac{1}{4} + \frac{1}{j2\pi} + \frac{1}{3 - j\frac{1}{\pi}}\right)} = 8.2477 + j1.7670 = 8.43\angle 12.1^\circ$$

$$i_0 = \frac{v}{3 - j\frac{1}{\pi}} = 2.6568 + j0.8709 = 2.8\angle 18.1^\circ$$

4: If $v_S(t) = -3\sin(100t + 45^\circ)$, find the value of $i_X(t)$ in phasor, rectangular, and time domain forms. *Hint: Convert the sine to an equivalent cosine with a positive amplitude and work from there.*



$$v_S(t) = -3\sin(100t + 45^\circ) = 3\cos(100t + 45^\circ \underbrace{-90^\circ}_{\text{sine in terms of cosine}} \underbrace{\pm 180^\circ}_{\text{sign}})$$

$= 3\cos(100t + 135^\circ)$

$\omega = 100,$ but the impedance conversions have already been done

Defining the currents to be clockwise.

Mesh 1:

$-v_S + 2i_1 + j2(i_1 - i_X) = 0$

$(2 + j2)i_1 - j2i_X = v_s$

Mesh 2:

$1\Omega i_X - j1i_X + j2(i_X - i_1) = 0$

$-j2i_1 + (1 - j1 + j2)i_X = 0$

In matrix form

$$\begin{bmatrix} 2+j2 & -j2 \\ -j2 & 1+j1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_X \end{bmatrix} = \begin{bmatrix} 3\angle 135^\circ \\ 0 \end{bmatrix} = \begin{bmatrix} \text{-2.1213 + 2.1213i} \\ \text{0.0000 + 0.0000i} \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ i_X \end{bmatrix} = \begin{bmatrix} \text{-0.5303 + 0.5303i} \\ \text{-1.0607 + 0.0000i} \end{bmatrix}$$

$i_X = -1.067\text{A} = 1.067\angle 180^\circ \equiv 1.067\cos(100t \pm 180^\circ)$