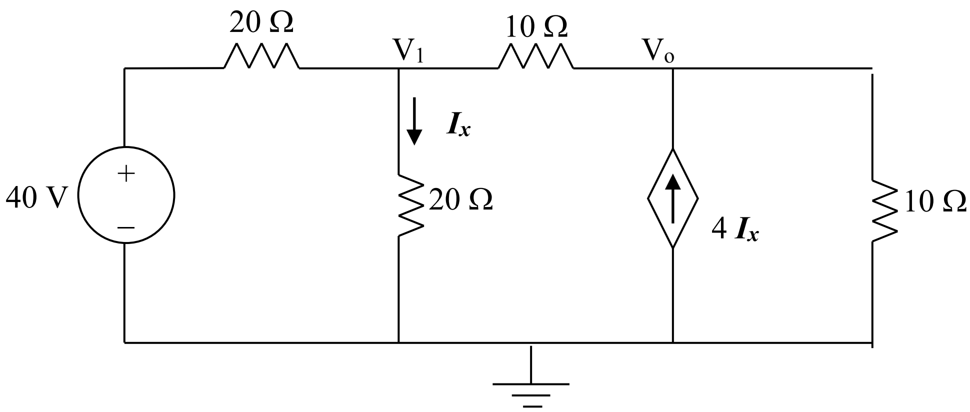
Solution 3.12

There are two unknown nodes, as shown in the circuit below.



At node 1,

$$\frac{V_1 - 40}{20} + \frac{V_1 - 0}{20} + \frac{V_1 - V_o}{10} = 0 \text{ or}$$

$$(0.05{+}0.05{+}.1)V_1 - 0.1V_o = 0.2V_1 - 0.1V_o = 2 \qquad (1)$$

At node o,

$$\frac{V_o - V_1}{10} - 4I_x + \frac{V_o - 0}{10} = 0 \text{ and } I_x = V_1/20$$

$$-0.1V_1 - 0.2V_1 + 0.2V_o = -0.3V_1 + 0.2V_o = 0 \text{ or} \qquad (2)$$

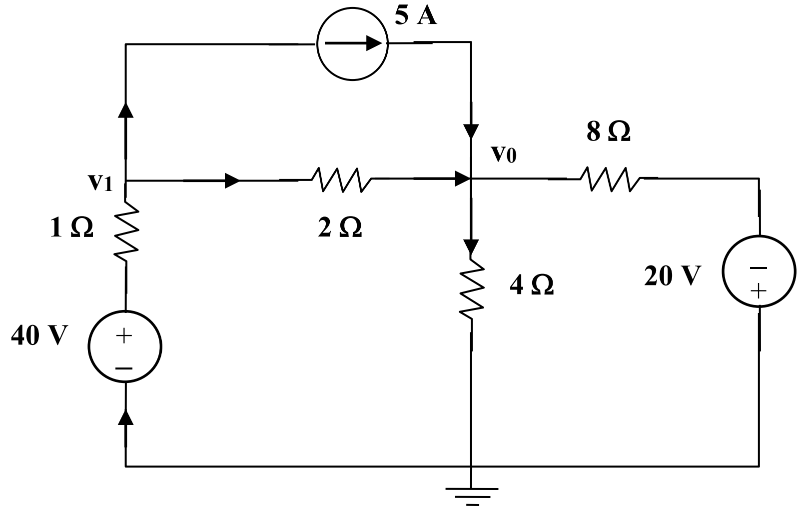
$$V_1 = (2/3)V_o \qquad (3)$$

Substituting (3) into (1),

$$0.2(2/3)V_o - 0.1V_o = 0.03333V_o = 2 \text{ or}$$

$$V_o = \mathbf{60\ V}.$$

Solution 3.15



Nodes 1 and 2 form a supernode so that $v_1 = v_2 + 10$ (1)

At the supernode, $2 + 6v_1 + 5v_2 = 3(v_3 - v_2) \longrightarrow \quad 2 + 6v_1 + 8v_2 = 3v_3$ (2)

At node 3, $2 + 4 = 3(v_3 - v_2) \longrightarrow \quad v_3 = v_2 + 2$ (3)

Substituting (1) and (3) into (2),

$$2 + 6v_2 + 60 + 8v_2 = 3v_2 + 6 \longrightarrow v_2 = \frac{-56}{11}$$

$$v_1 = v_2 + 10 = \frac{54}{11}$$

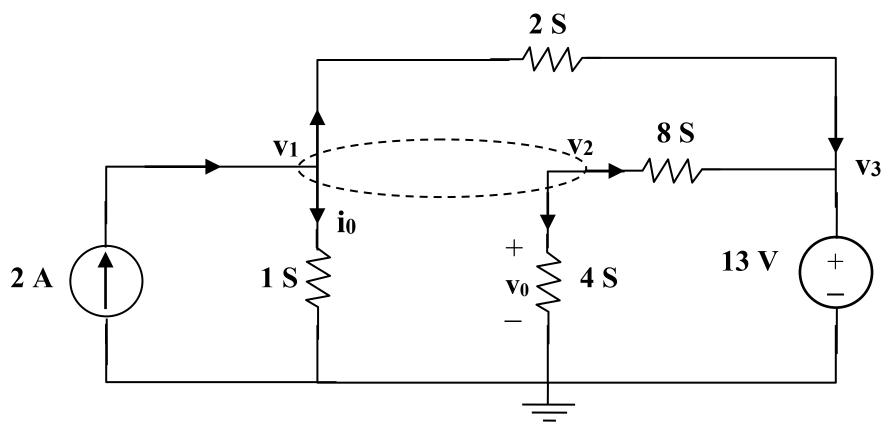
$$i_0 = 6v_i = \mathbf{29.45\ A}$$

$$P_{65} = \frac{v_1^2}{R} = v_1^2 G = \left(\frac{54}{11}\right)^2 6 = \mathbf{144.6\ W}$$

$$P_{55} = v_2^2 G = \left(\frac{-56}{11}\right)^2 5 = \mathbf{129.6\ W}$$

$$P_{35} = (v_L - v_3)^2 G = (2)^2 3 = \mathbf{12\ W}$$

Solution 3.16



At the supernode,

$$2 = v_1 + 2\,(v_1 - v_3) + 8(v_2 - v_3) + 4v_2, \text{ which leads to } 2 = 3v_1 + 12v_2 - 10v_3 \quad (1)$$

But

$v_1 = v_2 + 2v_0$ and $v_0 = v_2$.

Hence

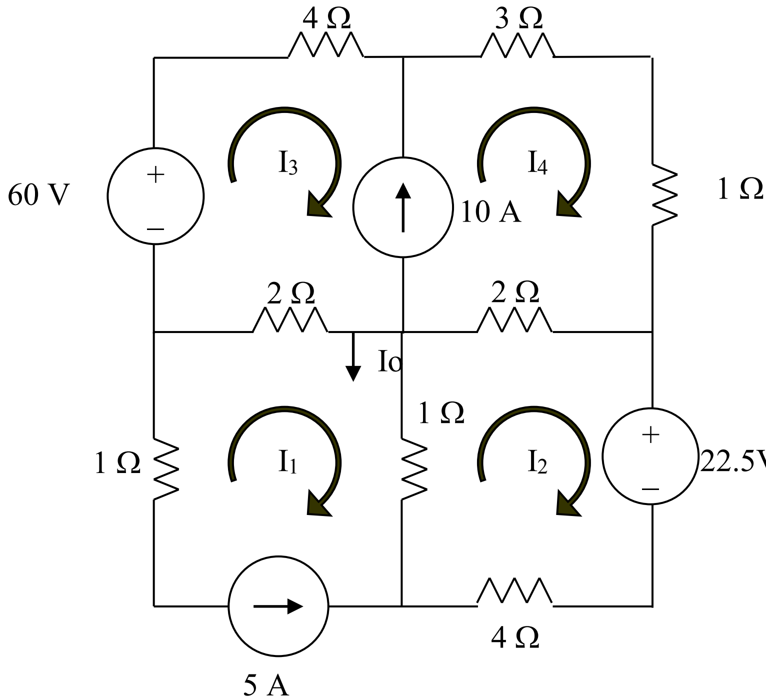
$$v_1 = 3v_2 \qquad (2)$$
$$v_3 = 13V \qquad (3)$$

Substituting (2) and (3) with (1) gives,

$v_1 =$ **18.858 V**, $v_2 =$ **6.286 V**, $v_3 =$ **13 V**

Solution 3.38

Consider the circuit below with the mesh currents.



$$I_1 = -5 \text{ A} \qquad (1)$$

$$1(I_2 - I_1) + 2(I_2 - I_4) + 22.5 + 4I_2 = 0$$
$$7I_2 - I_4 = -27.5 \qquad (2)$$

$$-60 + 4I_3 + 3I_4 + 1I_4 + 2(I_4 - I_2) + 2(I_3 - I_1) = 0 \text{ (super mesh)}$$
$$-2I_2 + 6\,I_3 + 6I_4 = +60 - 10 = 50 \qquad (3)$$

But, we need one more equation, so we use the constraint equation $-I_3 + I_4 = 10$. This now gives us three equations with three unknowns.

$$\begin{bmatrix} 7 & 0 & -1 \\ -2 & 6 & 6 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} -27.5 \\ 50 \\ 10 \end{bmatrix}$$

We can now use MATLAB to solve the problem.

```
>> Z=[7,0,-1;-2,6,6;0,-1,0]
```

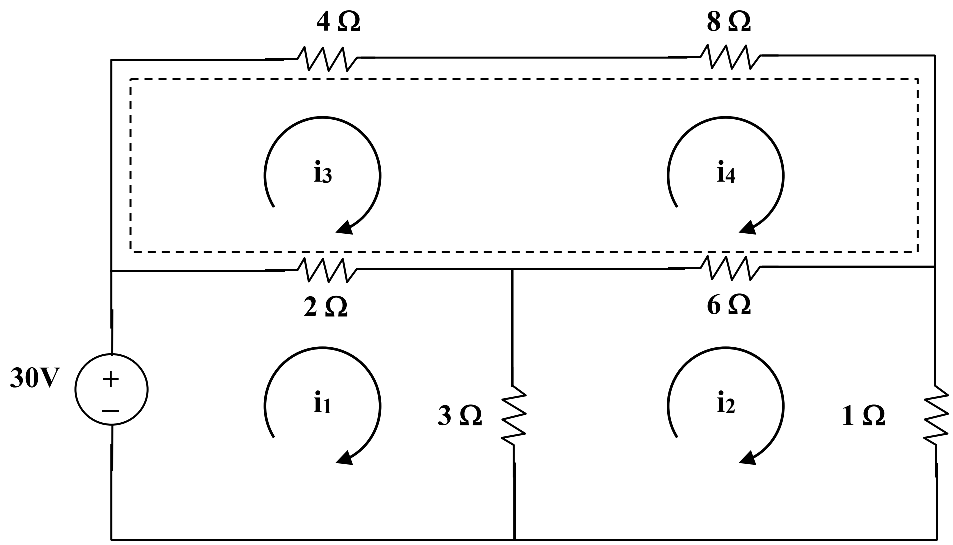
```
Z =

     7     0    -1
    -2     6     6
     0    -1     0
>> V=[–27.5,50,10]'

V =
   –27.5
    50
    10
>> I=inv(Z)*V

I =
   –1.3750
   –10.0000
    17.8750
```

$I_o = I_1 - I_2 = -5 - 1.375 =$ **–6.375 A**.

Check using the super mesh (equation (3)):

$-2I_2 + 6\,I_3 + 6I_4 = 2.75 - 60 + 107.25 = 50!$

Solution 3.45



For loop 1, $30 = 5i_1 - 3i_2 - 2i_3$ (1)

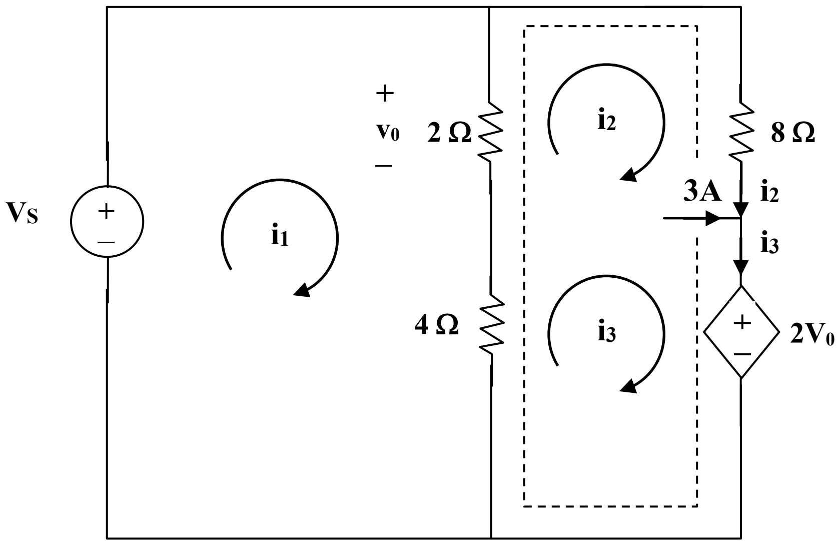
For loop 2, $10i_2 - 3i_1 - 6i_4 = 0$ (2)

For the supermesh, $6i_3 + 14i_4 - 2i_1 - 6i_2 = 0$ (3)

But $i_4 - i_3 = 4$ which leads to $i_4 = i_3 + 4$ (4)

Solving (1) to (4) by elimination gives $i = i_1 =$ **8.561 A**.

Solution 3.52



For mesh 1,

$$2(i_1 - i_2) + 4(i_1 - i_3) - 12 = 0 \quad \text{which leads to} \quad 3i_1 - i_2 - 2i_3 = 6 \qquad (1)$$

For the supermesh, $2(i_2 - i_1) + 8i_2 + 2v_0 + 4(i_3 - i_1) = 0$

But $v_0 = 2(i_1 - i_2)$ which leads to $-i_1 + 3i_2 + 2i_3 = 0$
(2)

For the independent current source, $i_3 = 3 + i_2$ (3)

Solving (1), (2), and (3), we obtain,

i_1 = **3.5 A**, i_2 = **-0.5 A**, i_3 = **2.5 A**.