Notes for CMPE306 Fall 2019 Exam #2

Capacitors

The voltage across a capacitor may not change abruptly. $i_C(t) = C\dfrac{dv_C(t)}{dt}$, $v_C(t) = \dfrac{1}{C}\int_0^t i_C(t)\,dt + v_C(0)$, $Z_C = \dfrac{1}{j\omega C}$

Inductors

The current through an inductor may not change abruptly. $v_L(t) = L\dfrac{di_L(t)}{dt}$, $i_L(t) = \dfrac{1}{L}\int_0^t v_L(t)\,dt + i_L(0)$, $Z_L = j\omega L$

The unit step and unit impulse functions:

$u(t) = 1$ whenever the argument $t > 0$; and is zero otherwise; $\int_{-\varepsilon}^{\varepsilon} \delta(t)dt = 1$.

$\int_{-\infty}^{t} \delta(t)\,dt = u(t)$; $\dfrac{du(t)}{dt} = \delta(t)$

First order differential equations

$\dfrac{dx}{dt} + ax = X$; Solution is $x(t) = x_t(t) + x_{SS}(t)$, where $x_{SS}(t) = x(\infty)$, $x(t) = [x(0) - x(\infty)]e^{-t/\tau} + x(\infty)$. τ is called the time constant.

Second order differential equations

The solution is again $x(t) = x_T(t) + x_{SS}(t)$, where the steady state depends only on source and the transient depends only on the initial conditions.

$\dfrac{d^2x}{dt^2} + a\dfrac{dx}{dt} + bx = \dfrac{X_S}{c}$, where $x(t)$ is either the current through the inductor ($i_L(t)$), or the voltage across the capacitor ($v_C(t)$)

Characteristic Equation: $s^2 + as + b = 0$ with solutions $s = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$

The solution is $x(t) = x_t(t) + x_{SS}(t)$ $x_{SS}(t)$ is the steady state solution, i.e., $x(\infty)$.

If $\alpha > \omega_0$, over damped, $x_t(t) = A_1e^{s_1t} + A_2e^{s_2t}$,

if $\alpha = \omega_0$, critically damped, $x_t(t) = (A_1t + A_2)e^{st}$

If $\alpha < \omega_0$, under damped, $x_t(t) = A_1e^{-\alpha t}\cos(\sqrt{\omega_0^2 - \alpha^2}\,t) + A_2e^{-\alpha t}\sin(\sqrt{\omega_0^2 - \alpha^2}\,t)$

A_1 and A_2 are determined using the initial conditions $x(t)\big|_{t=0^+}$, $\dfrac{dx}{dt}\Big|_{t=0^+}$, using the **total solution**, $x(t) = x_T(t) + x_{SS}(t)$!

R-L-C Circuits

Series RLC: $\dfrac{d^2x}{dt^2} + \dfrac{R}{L}\dfrac{dx}{dt} + \dfrac{x}{LC} = 0$, Parallel RLC: $\dfrac{d^2x}{dt^2} + \dfrac{1}{RC}\dfrac{dx}{dt} + \dfrac{x}{LC} = 0$, solution gives form of $x_T(t)$

Series RLC $\alpha = \dfrac{R}{2L}$ $\omega_0 = \sqrt{\dfrac{1}{LC}}$ **Parallel RLC** $\alpha = \dfrac{1}{2RC}$ $\omega_0 = \sqrt{\dfrac{1}{LC}}$

Ideal Op Amps

The voltage at one input to an ideal op amp is equal to the voltage at the other input.
The current on one input to an ideal op amp is equal to the current on the other input, and both are equal to zero.

Inverting amplifier: $\mathbf{V}_0 / \mathbf{V}_s = -\mathbf{Z}_f / \mathbf{Z}_i$, where $\mathbf{Z}_f, \mathbf{Z}_i$ are the feedback and input impedances, respectively

CMPE306 Fall 2019, Exam #2

Non-inverting amplifier $\mathbf{V}_0 / \mathbf{V}_i = 1 + \mathbf{Z}_f / \mathbf{Z}_i$, where $\mathbf{Z}_f, \mathbf{Z}_i$ are the feedback and input impedances, respectively

CMPE306 Fall 2019, Exam #2

Complex numbers & Phasors

Complex manipulations: Phasor notation $A\cos(\omega t+\phi) = A\angle\phi$.

We reference everthing to $\cos(\omega t+0^\circ)$, $\sin(\theta)=\cos(\theta-90^\circ)$

Add complex or phasor values using the rectangular form $\mathbf{X} = A\angle\phi$ is equivalent to $A\cos(\omega t+\phi)+jA\sin(\omega t+\phi)$.

$$\mathbf{X}+\mathbf{Y} = A\cos(\omega t+\phi)+jA\sin\left(\omega t+\phi\right)+B\cos\left(\omega t+\theta\right)+jB\sin(\omega t+\theta)$$

$$=\left(A\cos(\omega t+\phi)+B\cos\left(\omega t+\theta\right)\right)+j\left(A\sin(\omega t+\phi)+B\cos\left(\omega t+\theta\right)\right)$$

Multiply complex or phasor values using the phasor form $\mathbf{XY} = \left(A\angle\phi\right)\left(B\angle\theta\right) = AB\angle\phi+\theta$

Divide complex or phasor values using the phasor form $\mathbf{X}/\mathbf{Y} = \left(A\angle\phi\right)/\left(B\angle\theta\right) = \left(A/B\right)\angle\phi-\theta$

AC Circuits

Resistor: $\mathbf{V}=\mathbf{I}R$, Inductor $\mathbf{V}=\mathbf{I}\mathbf{Z}_L = j\omega L\mathbf{I}$, Capacitor $\mathbf{V}=\mathbf{I}\mathbf{Z}_C=\mathbf{I}/j\omega C$, $\omega=2\pi f$.

Thevenin, node, and mesh analysis work fine, using **Z** instead of **R.** Impedances add (in the complex sense) just like resistors.

Nodal analysis:
1) Identify the nodes, node voltages and all currents. Include the direction on all current flows. 2) Use Ohm's law to write expressions for resistor currents (the currents through resistors) in terms of node voltages wherever possible. 3) Use KCL to write the expressions for currents at all nodes or supernodes. 4) If necessary, use KVL to write expressions for the node voltages to provide one independent equation for each unknown node voltage. 4) solve the equations for the node voltages. 5) use the node voltages to solve for any necessary currents.

Mesh analysis:
1) Identify a current for each mesh in the circuit. Use a common direction (clockwise) for all mesh current definitions. 2) Use Ohm's law and KVL to write the equations for each mesh in terms of the unknown mesh currents. 3) solve the equations for the mesh currents. 4) use the mesh currents to solve for any circuit currents or node voltages.

Superposition: The voltage across any element is equal to the sum of the voltages generated by the independent voltage and current sources taken one at a time, with the other sources turned off.

Source Transformation: A voltage source in series with a resistor (impedance) may be transformed into a current source with the same resistance (impedance) in parallel by means of Ohm's law: $V_S = I_S\mathbf{Z}$.

Norton's Theorem: Any linear two port circuit network may be replaced by an equivalent current source with an equivalent resistance in parallel.

$I_N =$ short circuit current, $\mathbf{Z}_N$ = input resistance (impedance) with all independent sources turned off

Thevenin's Theorem: Any linear two port circuit network may be replaced by an equivalent voltage source with an equivalent resistance in series.

$V_{TH} =$ open circuit voltage, $\mathbf{Z}_{TH}$ = input resistance (impedance) with all independent sources turned off