Assignments:
Farum: Units & First Order (30 pts)
Fahmida: Complex Impedance (25 pts)
Hossain: Op Amp (30 pts)
EFCL: LRC IC and Soln's (35 pts)

General grading principles:

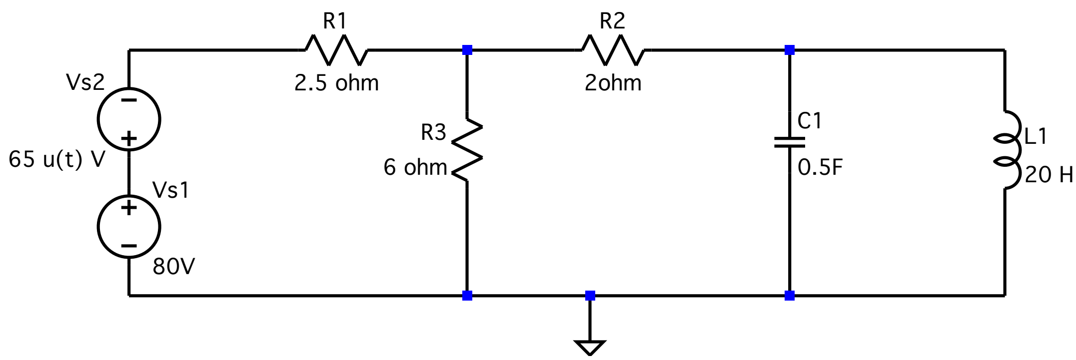
I'm far more concerned with the circuits information and process than the actual arithmetic. You will see that the detailed rubrics convey that in terms of how partial credit is awarded. If you are unsure, please ask me!

Within your grading assignment, please follow two rules. 1) Grade all of one problem and then all of the other (if you have two) and 2) be as consistent as possible. If the rubric isn't precise and you decide to deduct a point because the algebraic sign isn't correct, then you should make the same decision for ALL students.

1. (10 pts) Short Answer. In the open column, *write down the term from the Column B* that most appropriately matches with the term in Column A. There will be a penalty for not following instructions! **Not all items from Column B will be used!**

Mho-second or Sieman-second	*Capacitance or Farad*	Angular frequency
$2\pi f$	*Angular frequency*	Admittance
Hertz	*Circular Frequency*	Circular frequency
Ampere-second/volt	*Capacitance or Farad*	Capacitance or Farad
Unit of $\mathbf{Z}_C = 1/j\omega C$	*Resistance or Ohm*	Inductance or Henry
$\sec^{-1}$	*Circular frequency*	Reactance
Volt-second/ampere	*Inductance or Henry*	Resistance or Ohm
Rad/sec	*Angular frequency*	Furlongs per fortnight
Ohm-second	*Inductance or Henry*	Parsecs
Imaginary part of impedance	*Reactance*	Power spectral flux density

2. **(18 points) LRC Initial Conditions #2 (Homework 7, Problem 4)** Find six initial and final conditions necessary to solve for $i_L(t)$, $v_C(t)$ for $t > 0$. *Hint: What changes in the circuit, and what effect does it have?*



Grading rubric
3 points for each correct value. Three significant figures are acceptable.
Partial credit
For each value, award up to 2 points for proper application of circuit principles. Award 1 point for proper computation.

Solving for $i_L(t), t>0,$ we need $i_L(0^+), i_L(\infty), \left.\frac{di_L}{dt}\right|_{t=0+}$

for $v_C(t), t>0,$ we need $v_C(0^+), v_C(\infty), \left.\frac{dv_C}{dt}\right|_{t=0+}$

At 0^-, 65V source is off ($u(t)=0,$ per notes sheet), capacitor is open, inductor is short. Network is $2.5\Omega + 6\Omega \,\|\, 2\Omega = 2.5 + \frac{6\times 2}{8} = 4\Omega.$

Current across $R1$ is $\frac{80\text{V}}{4\Omega} = 20\text{A}.$

$\boxed{v_C(0^-) = v_C(0^+) = 0\text{V}}$, because capacitor is in parallel with inductor (short).

$6\Omega \,\|\, 2\Omega$ is a current divider, $\boxed{i_L(0^-) = 20\text{A} \times \frac{6\Omega}{6\Omega + 2\Omega} = 15\text{A} = i_L(0^+)}$

At $t=\infty,$ $u(t)=1,$ 65V source is on and *subtracts* from 80V due to KVL.

$i_L(\infty) = \frac{15\text{V}}{4\Omega} \times \frac{6\Omega}{6\Omega + 2\Omega} = \frac{90}{32}\text{A}$

$\boxed{i_L(\infty) = \frac{45}{16}\text{A} = 2\frac{13}{16}\text{A} = 2.8125\text{A}}$

$\boxed{v_C(\infty) = 0\text{V}}$, as capacitor is in parallel with the inductor, and the inductor is a wire at steady-state.

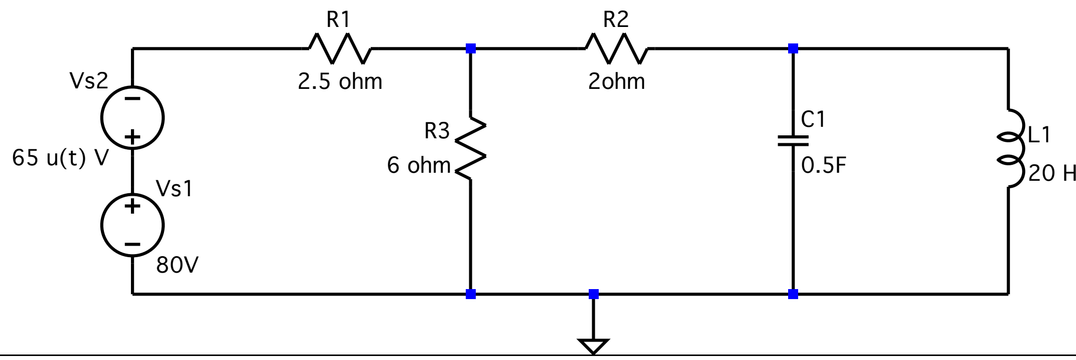
Immediately after the voltage changes from 80V to 15 V, $v_C(0^+) = 0$ held by the capacitor. $\left.\frac{di_L}{dt}\right|_{0+} = \frac{v_L(0^+)}{L} = 0\text{A/s}.$

Immediately after the voltage change, the current across R1 is $\frac{15\text{V}}{4\Omega} = \frac{15}{4}\text{A},$ which is then divided by the current divider so that the current through the 2Ω is $\frac{15}{4}\text{A} \times \frac{6\Omega}{6\Omega + 2\Omega} = \frac{45}{16}\text{A}.$ The inductor forces $i_L(0^+) = 15\text{A}.$

By KCL, $\frac{45}{16}\text{A} = i_C(0^+) + i_L(0^+),$ or $i_C(0^+) = \frac{45}{16} - 15 = -12.1875\text{A}$

$\boxed{v_C(0^+) = \frac{-12.1875}{0.5} = -24.375 \text{ V/s} = -24\frac{3}{8}\text{V/s}.}$

3. **(17 points) LRC Solution (Extended Homework 7 Problem 4)** Solve for $i_L(t)$, the current flowing *down* through the inductor in the previous circuit, reprinted below.



Note: If you were unable to obtain the initial conditions in Problem 5, you may follow the process for solving the second-order circuit for up to 80% of the credit (12 points). Explain what you are doing!

Grading Rubric

17 points for correct answer. Three significant figures are acceptable.

Partial credit. Up to 80% partial credit with erroneous or no solution to the previous problem. Award up to 80% (15 pts) partial credit if they have the wrong type of solution (critically damped, overdamped) but the proper process and math.

1 point for parallel RLC
3 points for alpha
3 points for w0
1 point for underdamped
1 point for generic underdamped solution
3 points for solution for A using i(0) initial condition
3 points for solution for B using di/dt at 0 initial condition
2 point for specific solution

The equivalent resistance is $2+6\,\|\,2.5=2+\dfrac{6\times 2.5}{6+2.5}=3.76\Omega$

Parallel RLC

$$\alpha=\frac{1}{2RC}=\frac{1}{2(3.76\Omega)(0.5\text{F})}=0.2660\ /\text{s}$$

$$\omega_0=\frac{1}{\sqrt{LC}}=\frac{1}{\sqrt{20\text{H}\times 0.5\text{F}}}=\frac{1}{\sqrt{10}}=0.3162$$

$\omega_0>\alpha\Rightarrow$ underdamped

$$\omega_D=\sqrt{\omega_0^2-\alpha^2}=0.1711$$

$$i_L(t)=\left(A_1\cos\omega_D t+A_2\sin\omega_D t\right)e^{-\alpha t}+i_L(\infty)$$

$$i_L(0)=15\text{A}=\left(A_1\times 1+A_2\times 0\right)e^{-0}+2.8125\text{A}$$

$$A_1=12.1875$$

$$\frac{di_L}{dt}=\left(-A_1\omega_D\sin\omega_D t+A_2\omega_D\cos\omega_D t\right)e^{-\alpha t}+\left(A_1\cos\omega_D t+A_2\sin\omega_D t\right)e^{-\alpha t}(-\alpha)$$

$$\left.\frac{di_L}{dt}\right|_{t=0}=0=\left(A_1\times 0+A_2\omega_D\times 1\right)\times 1+\left(A_1\times 1+A_2\times 0\right)\times 1\times(-\alpha)$$

$$0=A_2\omega_D-A_1\alpha=0.1711A_2-12.1875(0.2660)$$

$$12.1875(0.2660)=0.1711A_2;\ \ A_2=18.9473$$

$$i_L(t)=\left(12.1875\cos 0.1711t+18.9473\sin 0.1711t\right)e^{-0.2660t}+2.8125\text{A}$$

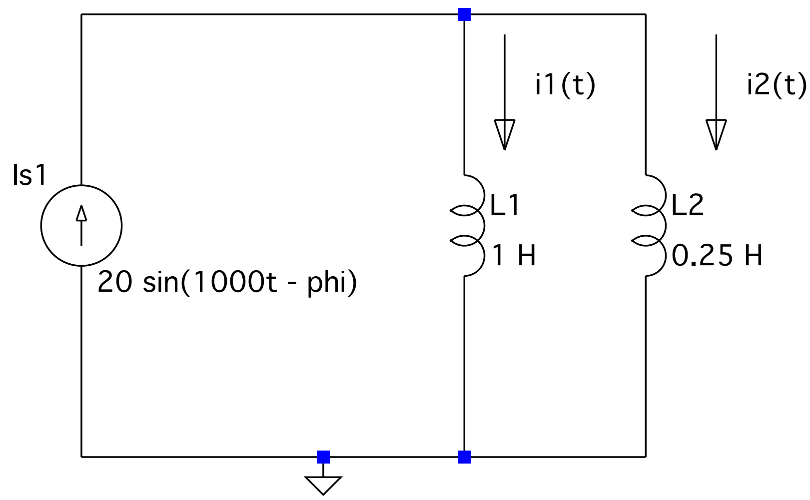
4. **Complex Impedance and Phasors (25 pts)** For this simple AC circuit, write expressions for the following. The source phase is $\phi = 45^{\circ}$.

4.1 Express the current source as a phasor in standard form (relative to a cosine function).

4.2 Transform the phasor source to rectangular (real and imaginary) form.

4.3 Write the complex impedances of inductors L1 and L2.

4.4 Determine the phasor representation of the currents i1 and i2.



4.5 Determine the phasor representation of the voltage across L1.

Grading Rubric

5 points for each solution (5 x 5 = 20). For 4.3 and 4.4, award 2.5 points for each correct answer

Up to 3 points partial credit at grader discretion

If 4.1 is wrong, but 4.2, 4.3 and 4.4 correctly use the erroneous 4.1 value award 4 pts for each.

4.1 $20\sin(1000t - 45^{\circ}) = 20\cos(1000t - 45^{\circ} - 90^{\circ}) = 20\cos(1000t - 135^{\circ})$

$\mathbf{i}_s = 20\angle -135^{\circ}$

4.2 $\mathbf{i}_s = 20\angle -135^{\circ} = 20\cos(-135^{\circ}) + j20\sin(-135^{\circ}) = -14.1 - j14.1$

4.3 $\mathbf{Z}_1 = j\omega L_1 = j(1000)(1\text{H}) = j1000\Omega$

$\mathbf{Z}_2 = j\omega L_2 = j(1000)(0.25\text{H}) = j250\Omega$

4.4 Parallel impedances are a current divider

$$\mathbf{i}_1 = \mathbf{i}_s \times \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} = 20\angle -135^{\circ} \times \frac{j250}{j1000 + j250} = 20\angle -135^{\circ} \times \left(\frac{j}{j}\right)\left(\frac{250}{1250}\right)$$

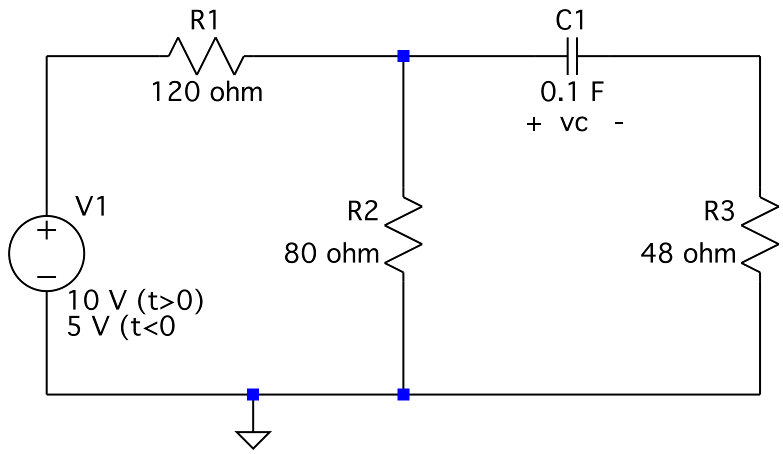
$= 4\angle -135^{\circ}$

$$\mathbf{i}_2 = \mathbf{i}_s \times \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2} = 16\angle -135^{\circ} \quad \text{(also by KCL)}$$

4.5 $\mathbf{v} = \mathbf{i}_1 \times \mathbf{Z}_1 = 4\angle -135^{\circ} \times j1000\Omega = 4\angle -135^{\circ} \times 1000\angle 90^{\circ} = 4000\angle -45^{\circ}$

or $\mathbf{v} = \mathbf{i}_2 \times \mathbf{Z}_2 = 16\angle -135^{\circ} \times j250\Omega = 4000\angle -45^{\circ}$

5. **(20 points) First Order Response #1**
Find the voltage across the capacitor for $t > 0$ in the following circuit. *Hint: Look at the definition for V1 very carefully!*



20 pts for correct solution

Partial credit
4 pts for v(0)
4 pts for v(inf)
4 pts for proper equivalent resistance
4 pts for tau (award 2 for correct process with erroneous resistance)
1 pt for general solution (award this point if the specific solution is correct)
3 pts for specific solution

First order, we need $v(0^+), v(\infty), \tau$

$t = 0^-, V_1 = 5\text{V}$, capacitor is open, so there is no drop across 48 Ω

120Ω and 80Ω voltage divider,

$$v_c(0^-) = v_{80\Omega} = 5\text{V} \times \frac{80}{80+120} = 2\text{V}.$$

$t = \infty$, capacitor again open

$$v(\infty) = 10\text{V} \times \frac{80}{80+120} = 4\text{V}.$$

For τ, find the equivalent resistance

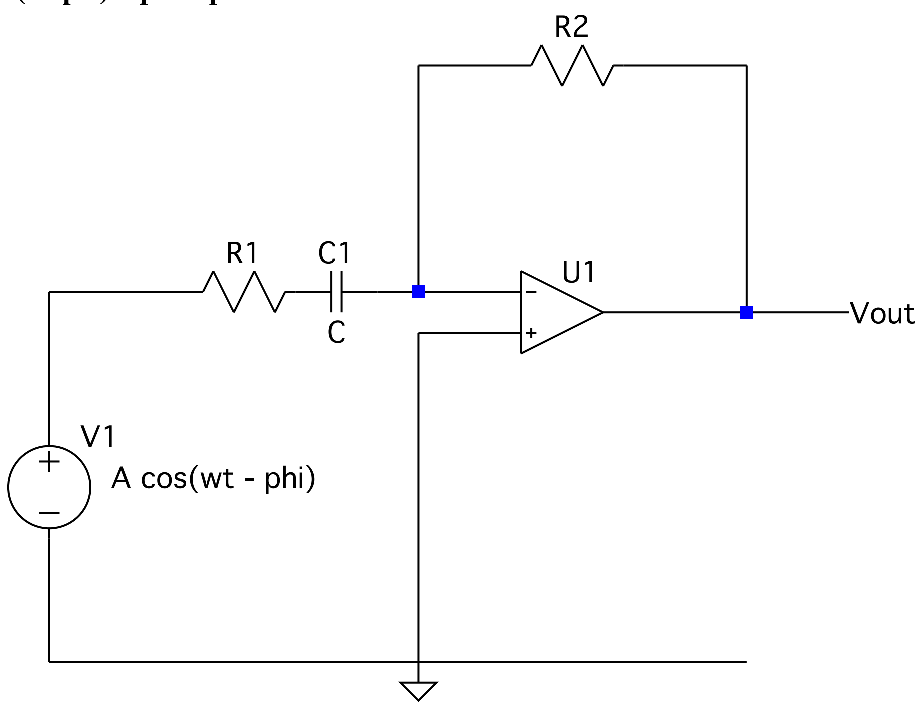
$$R_{EQ} = 48 + 120 \,\|\, 80 = 48 + \frac{120 \times 80}{120+80} = 96\Omega$$

$$\tau = RC = 96\Omega \times 0.1\text{F} = 9.6\text{s}$$

$$v(t) = v(\infty) + \left(v(0) - v(\infty)\right)e^{-t/\tau}$$

$$v(t) = 4 + \left(2 - 4\right)e^{-t/9.6} = 4 - 2e^{-t/9.6}, t > 0$$

6. **(25 pts) Op Amp LR/LC Circuit**



6.1 Express the input impedance $\mathbf{Z}_i$ as a function of the angular frequency of the source, ω.

Grading rubric

5 points for correct *input* impedance, Zi, or its algebraic equivalent

No partial credit on this one, but be careful to award points for algebraic equivalence.

$$\mathbf{Z}_i = R_1 + \frac{1}{j\omega C_1} = R_1 - \frac{j}{\omega C_1} = \frac{1 + j\omega R_1 C_1}{j\omega C_1}$$ or algebraic equivalent

6.2 Then write an expression for the (complex) voltage gain $G(\omega) = \dfrac{\mathbf{V}_0(\omega)}{\mathbf{V}_1(\omega)}$. Simplify the expression to phasor form. *Hint: This is not a time domain or transient response problem! This is a complex impedance problem.*

Grading rubric

10 points for correct answer or algebraic equivalent

Partial Credit

Award 8 points for G(w) as shown in the second line of my solution, or its algebraic equivalent. Be careful to assess equivalence and award these points if earned.

Award an additional 1 point for the correct magnitude or algebraic equivalent.

Award an additional 1 point for the correct phase or algebraic equivalent.

Award up to 5 points at your discretion for the correct process, but some erroneous math.

$$\mathbf{Z}_i = R_1 + \frac{1}{j\omega C_1} = R_1 - j\frac{1}{\omega C_1} = \frac{1 + j\omega R_1 C_1}{j\omega C_1}$$

$$\mathbf{G}(\omega) = -\frac{\mathbf{Z}_f}{\mathbf{Z}_i} = \frac{-R_2}{\left(\frac{1 + j\omega R_1 C_1}{j\omega C_1}\right)} = \frac{-j\omega R_2 C_1}{1 + j\omega R_1 C}$$

$$\left|\mathbf{G}(\omega)\right|^2 = \frac{|j\omega R_2 C|^2}{|1 + j\omega R_1 C|^2} = \frac{(\omega R_2 C)^2}{1 + \omega^2 R_1^2 C^2}$$

$$\angle \mathbf{G}(\omega) = 90^\circ - 180^\circ - \tan^{-1}\left(\omega R_1 C_1\right) = -90^\circ - \tan^{-1}\left(\omega R_1 C_1\right)$$

$$\left|\mathbf{G}(\omega)\right| = \sqrt{\frac{(\omega R_2 C)^2}{1 + \omega^2 R_1^2 C^2}}$$

$$\mathbf{G}\left(\omega\right) = \frac{\omega R_2 C}{\sqrt{1 + \omega^2 R_1^2 C^2}} \angle\left(-90^\circ - \tan^{-1}\left(\omega R_1 C_1\right)\right)$$

6.3 If the input voltage V1 is $10\sin\left(10t + \frac{\pi}{2}\right)$, $R_1 = 10\text{k}\Omega$, $R_2 = 5\text{k}\Omega$, $C = 10\mu\text{F}$, what is the steady state output voltage as a function of time? *Hint: This is not a time domain or transient response problem! This is a complex impedance problem.*

Grading Rubric:
10 pts for correct answer or algebraic equivalent (including real & imaginary parts)

Partial 2 pts for conversion to cosine (degress or radians)
6 pts for computation of Gain from 6.2
2 points for any correct value of output voltage in any equivalent form

Award 4 points for correct computation of Gain using incorrect gain value from 6.2

Award 1 pt for correct value application of gain if cosine conversion was incorrect.

$$10\sin\left(10t + \frac{\pi}{2}\right) = 10\cos(10t + \frac{\pi}{2} - \frac{\pi}{2}) = 10\cos(10t) \text{ or } 10\cos(10t) = 10\angle 0^\circ$$

Using final result from 6.2 above

$$\mathbf{G}\left(\omega\right) = \frac{\omega R_2 C}{\sqrt{1 + \omega^2 R_1^2 C^2}} \angle\left(-90^\circ - \tan^{-1}\left(\omega R_1 C_1\right)\right)$$

$$\mathbf{G}(10) = \frac{(10\text{ r/s})(5\text{k}\Omega)(10\mu\text{F})}{\sqrt{1 + (10\text{ r/s})^2 (10\text{k}\Omega)^2 (10\mu\text{F})^2}} \angle\left(-90^\circ - \tan^{-1}\left((10\text{ r/s})(10\text{k}\Omega)(10\mu\text{F})\right)\right)$$

$$= \frac{0.5}{\sqrt{1 + 1^2}} \angle(-90^\circ - \tan^{-1}(1)) = 0.3535\angle -135^\circ = 0.3535\angle -\frac{3\pi}{4}$$

$$\mathbf{V}_o = \mathbf{G}(10)\mathbf{V}_{in} = \left(0.3535\angle -\frac{3\pi}{4}\right)\left(10\angle 0^\circ\right) = 3.535\angle -\frac{3\pi}{4} = 3.535\text{V}\angle -\frac{3\pi}{4}$$

$$= 3.535\text{V}\angle -135^\circ = -2.5 - j2.5\text{V}$$